Shock wave propagation verification test of the Uintah MPM code

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1 Introduction

This document outlines a shock wave propagation problem that is used as a verification test for the Uintah material point method (MPM) code. The problem consists of a thin copper flyer plate impacting an aluminum target. The flyer plate thickness is 0.5 mm, and its initial velocity is 1 km/sec. These two materials were chosen because, as will be explained, when the flyer plate has a higher shock impedance than the target, the resulting wave structure in the target becomes much more complex than for a low impedance flyer. The Uintah simulations were performed using the CPDI interpolator, a mesh resolution of 0.01 mm, and artificial viscosity. The artificial viscosity coefficients were found to have a dramatic effect upon the simulation results.

2 Analytical solution

The pressure/particle velocity Hugoniot for the two materials is used throughout this document. The Hugoniot represents the locus of possible shocked states for a given material. The Hugoniot for these materials is derived from two equations: one empirical and one resulting from conservation of momentum. The empirical equation relates the shock velocity to the change in particle velocity across the shock. This empirical expression is

$$U_s = C_o + S(u_p - u_o) \tag{1}$$

where U_s is the shock velocity, u_p is the particle velocity behind the shock, and u_o is the particle velocity ahead of the shock. C_o and S are material constants which are measured in the laboratory. This simple formula accurately describes the shock behavior of a wide variety of materials, including copper and aluminum. The material constants for copper and aluminum are $C_o = 3940$ m/s and $S = 1.489$, and $C_o = 5386$ m/s and $S = 1.339$ respectively. The Mie Gruneisen equation of state, which is used in the Uintah simulations, is designed to produce this linear relationship between the change in particle velocity and the shock velocity.

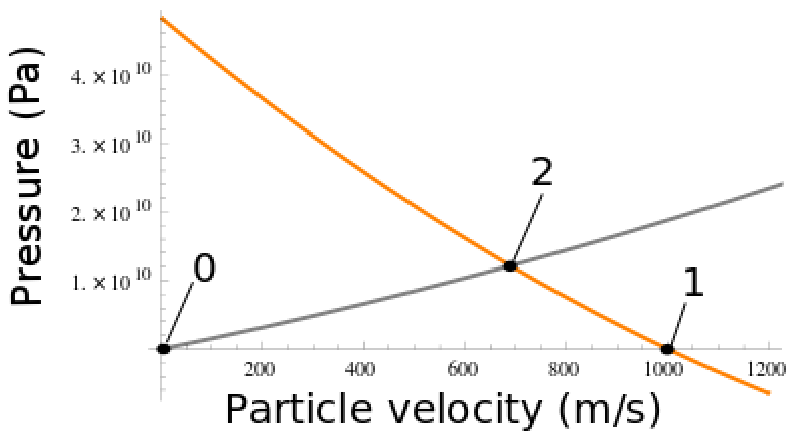

Figure 1: Pressure/particle velocity Hugoniot curves for copper (orange) and aluminum (gray). The copper curve represents the possible shocked material states for a left-going shock with an initial particle velocity of 1000 m/s.

The other governing equation for this problem is the momentum equation written for shocks. This is known as the momentum jump condition:

$$P - P_o = \rho_o(U_s - u_o)(u_p - u_o), \tag{2}$$

where ρ_o is the initial mass density, and P and P_o are the pressure in front of and behind the shock respectively. Equations 1 and 2 can be combined to form the Pressure/particle velocity Hugoniot:

$$P = P_o + (C_o + S(u_p - u_o))(u_p - u_o) \tag{3}$$

As written, this expression is only valid for right-going shocks. In order to describe left-going shocks the curve must be reflected about u_o.

2.1 Initial impact

Figure 1 shows the P/u_p Hugoniot curves for the initial impact. The aluminum target has an initial velocity of zero, and initial pressure of zero, which is labeled as state 0 in Figure 1. The copper flyer plate has an initial velocity of 1000 m/s and zero pressure, which is labeled as state 1 in Figure 1. The impact results in a right-going shock into the target and a left-going shock into the flyer plate. The pressure and particle velocity behind these shock waves will be the same in both materials. Therefore, the state behind the shock waves is determined by the intersection of the two Hugoniot curves in Figure 1. The resulting particle velocity is 690 m/s, and the pressure is $1.21e10$Pa, which is labeled as state 2 in Figure 1.

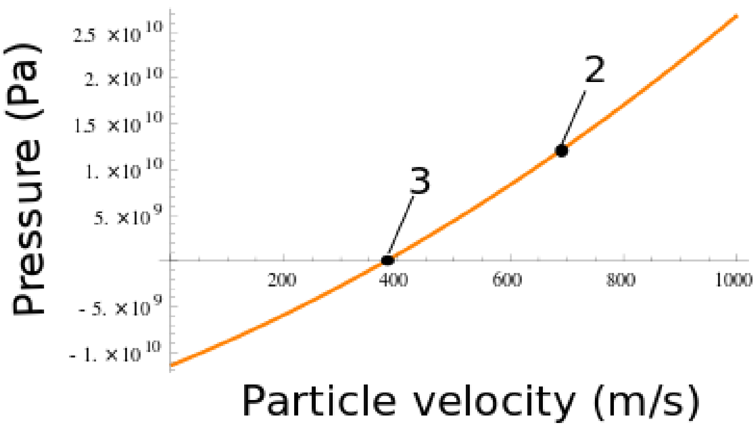

Figure 2: Pressure/particle velocity Hugoniot curve for a right-going shock in copper. The black dot indicates state 2, which is what resulted from the initial impact. The intersection of this curve with the zero-pressure axis is the particle velocity behind the rarefaction wave that propagates back toward the interface between the two materials.

2.2 First free-surface interaction

The left-going shock wave in the flyer-plate propagates through the flyer plate until it reaches the free-surface at the back of the plate. A right-going rarefaction wave will be produced as a result of the free-surface interaction. This rarefaction wave will be approximated as a shock. For small strains this is a reasonable assumptions since the isentrope (along which actual unloading takes places) is very close to the Hugoniot. Additionally, unlike the shock used to approximate it, the actual rarefaction wave will spread out as it propagates. To approximate the rarefaction as a shock, we need to find the zero-pressure intersection of the right-going copper Hugoniot which also passes through state 2. This is illustrated in Figure 2.

The resulting particle velocity is 380 m/s, and the pressure is zero. We will call this state 3. Note that if the shock impedance (slope of the secant line between initial and final states) of the flyer plate material were less than that of the target, the particle velocity after the free-surface interaction would be negative.

2.3 Second interface interaction

When the rarefaction wave in the flyer plate reaches the interface between the two materials a left-going wave will propagate back into the copper flyer plate, and a right-going wave will propagate into the aluminum. As with the initial impact, the state resulting from the second interface reaction is the intersection

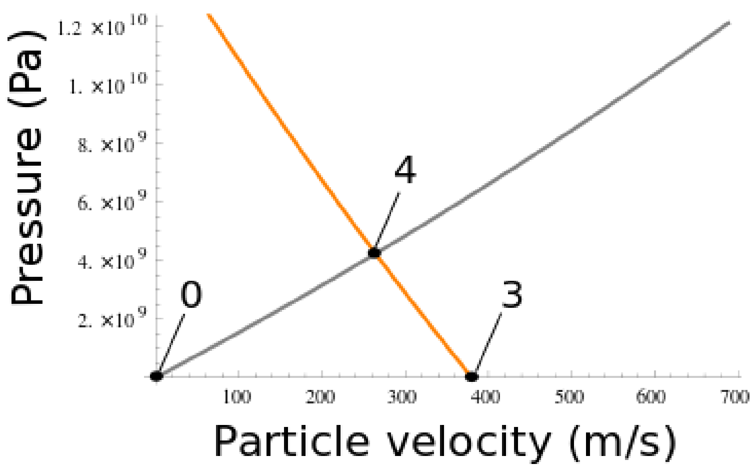

Figure 3: Pressure/particle velocity Hugoniot curves for copper (orange) and aluminum (gray). The curve for copper represents the possible shocked material states for a left-going shock with an initial particle velocity of 380 m/s.

of the right-going aluminum Hugoniot and the left-going copper Hugoniot. This time the copper Hugoniot must pass though state 3. The aluminum Hugoniot passes through state 2, which is just the same right-going Hugoniot which passes through the origin. These curves are shown in figure 3.

The resulting particle velocity is 264 m/s and the pressure is $4.2e9$ Pa, which is labeled as state 4 in Figure 3. This will result in a shock wave propagating to the left back into the flyer plate, and a rarefaction wave propagating to the right into the aluminum target. Note that if the flyer plate had a lower shock impedance than the target material, the particle velocity at state 2 would have been negative, and the intersection of the two Hugoniot curves for the second interface interaction would have been at a negative pressure. Since tension in the interface is impossible, this would cause the two materials to separate.

2.4 Second free-surface interaction and third interface interaction

The second free-surface interaction and third interface interaction behave much as the first interactions discussed in detail above. The second free surface interaction results in a rarefaction wave propagating to the right in the copper flyer plate. The particle velocity behind this rarefaction wave will be 149 m/s, and the pressure will be zero. This rarefaction wave will then reach the interface between the two materials and result in a shock wave propagating to the left into the copper flyer plate, and another rarefaction wave propagating to the right into the aluminum target. The particle velocity behind these two waves will be 104 m/s and the pressure will be $1.6e9$ Pa. This process will continue until the flyer plate comes to rest after the waves have completely dissipated. However, for our purposes no additional wave interactions will be considered.

3 Comparison with Uintah simulation

Figure 4 shows both the analytical solution and the numerical solution found using Uintah. The numerical simulation uses artificial viscosity to damp out the ringing in the solution. Without artificial viscosity the numerical solution becomes very chaotic, with unrealistic high-frequency pressure fluctuations. This is common in most explicit time-integration numerical schemes. An undesirable consequence of artificial viscosity is that the shock fronts which are discontinuities in the analytical solution, are spread out in the numerical solution. This phenomenon does occur in real materials, but in this case it is a consequence of artificial viscosity rather than an a calibrated material viscosity. The numerical simulation was performed for various values of the two artificial viscosity coefficients available in Uintah. The default values are 0.2 for the linear term and 2.0 for the quadratic term. With these values the numerical solution was very smooth with no obvious steps in the wave structure being visible. As a result of the large artificial viscosity the wave rapidly dissipated. The artificial viscosity coefficients were gradually decreased until the solution become very noisy. As the artificial viscosity coefficients were decreased, the amplitude of the shock wave approached that of the analytical solution, and the shock fronts became increasingly steep. The steps in the wave structure also became visible with reduced artificial viscosity. The numerical solution shown in figure 4 used linear and quadratic artificial viscosity coefficients of 0.03 and 0.3 respectively. Note that in Figure 4 the rarefaction waves are in fact spreading out more than the shock wave. The first rarefaction wave is spread out to the point that it is beginning to reach the shock front and attenuate the shock. This phenomenon is known as hydrodynamic attenuation. It is not observed in the analytical solution since the rarefaction waves are treated as shock waves.

4 Conclusion

An impact/shock wave propagation problem was solved both analytically and numerically using the Uintah MPM code. The numerical and analytical solutions produced waves with nearly the same amplitude, and with a similar wave shape. The analytical solution approximated the rarefaction waves as shocks, which causes them to be unrealistically steep. The numerical solution required artificial viscosity to damp out high-frequency “ringing”, and this caused both the rarefaction waves and the shock waves to be spread out more than the rate-independent material models would predict without the artificial viscosity.

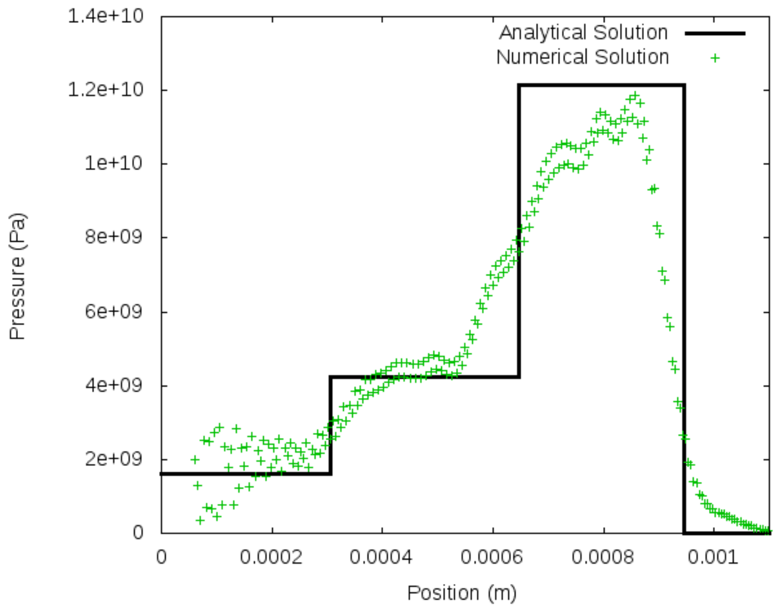

Figure 4: Pressure versus position in the target material for both the analytical solution (black line) and Uintah simulation (green marks). The analytical solution approximates rarefaction waves as shock waves. The Uintah simulation includes artificial viscosity which cause the shock fronts to become spread out.

Nearly all explicit time-integration schemes suffer from ringing instabilities in the absence of artificial viscosity, especially with extreme gradients in the solution as are present in shock waves. Considering these factors, the numerical and analytical solutions agree as well as can be hoped for with any solution obtained with explicit time-integration.